Importance Sampling of Area Lights in Participating Media

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(a) (b) (c) (d) (e)

Figure 1: *Point light source with density sampling (a) vs. our method (b). Textured area light with density sampling (c), our method (d), and our method with MIS (e). All images use 16 light samples per pixel.*

We focus on the problem of single scattering in homogeneous volumes and develop a new importance sampling technique that avoids the singularity near point light sources. We then generalize our method to area lights of arbitrary shapes.

Previous work in unbiased volume rendering [Yue et al. 2010] has concentrated on efficient importance sampling of the transmission term. In homogeneous media this term is non-zero and smoothly varying, thus the light sources are a much greater source of noise.

Importance sampling for point light sources The volume rendering equation for a single point light in a homogeneous medium can be expressed as:

$$L(x, \vec{\omega}) = \int_a^b \sigma_s e^{-\sigma_t (t + \Delta + \sqrt{D^2 + t^2})} \frac{\Phi}{D^2 + t^2} dt \qquad (1)$$

Figure 2 describes the involved parameters. Note that to simplify the notation, we re-parameterize t so that the origin is the orthogonal projection of the light onto the ray. This change modifies the integration bounds a and b (which can be negative now) and adds an extra term Δ which is the distance between the real origin and the new one.

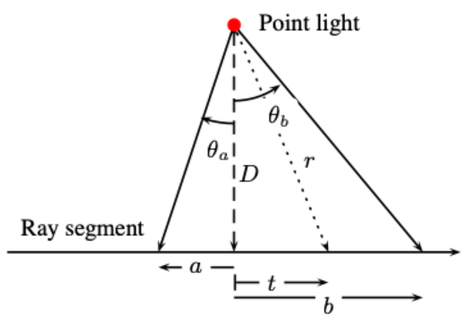

Figure 2: *Equi-angular sampling configuration*

Designing a PDF proportional to the $1/r^2$ term, we obtain the following normalized equation and sampling function (ξ in $[0, 1)$):

$$pdf(t) = \frac{D}{(\theta_b - \theta_a)(D^2 + t^2)} \qquad (2)$$

$$t(\xi) = D \tan\Big((1 - \xi)\theta_a + \xi\theta_b\Big) \qquad (3)$$

Equation 3 reveals that this technique makes equal angle steps along the ray. We thus refer to this technique as *equi-angular sampling*.

Importance sampling for area lights For area lights, the equation becomes more complex, as we have a nested integral over the surface of the light. Most notably, the distance D which was constant before is now a function of the position sampled from the light.

We could make a simplifying assumption and choose an arbitrary point like the center of the light to apply the previous equation. While this approximation works well for small light sources, it fails as the light becomes larger. The edges of the light source become noisier as the D^2 term in the PDF starts to dominate.

Our key insight is that we can distribute the error from the non-constant D by simply using the sample point we will estimate radiance from as our center for equations 2 and 3. While this does not solve the potential singularity caused by sometimes choosing a sampling center too close to the ray, the error is now uniformly distributed over the surface of the light, even in challenging cases like stretched rectangular lights, or textured area lights.

This remaining noise can be masked by applying multiple importance sampling [Veach and Guibas 1995] between the area light surface and the phase function. As our method has concentrated most high variance noise close to the light's surface, MIS can be much more effective than with other line sampling distributions (see supplemental material for examples).

Results Figure 1 shows some sample renderings performed with and without our method. Our new sampling equations substantially reduce variance. As our method is a generic sampling technique, it can be used to accelerate several light transport algorithms, including path tracing and bidirectional path tracing.

References

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